SPIN CURRENT NOISE AS A PROBE OF INTERACTIONS

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The spin resolved current shot noise can uniquely probe the interactions in mesoscopic systems: i) in a normal-superconducting junction, the spin current noise is zero, as carried by singlets, and ii) in a single electron transistor (SET) in the sequential regime, the spin current noise is Poissonian. Coulomb interactions lead to usually repulsive, but also attractive correlations. Spin current shot noise can also be used to measure the spin relaxation time T_1.

Non-equilibrium (shot) noise provides information about the charge and the statistics of carriers in mesoscopic systems [1]. The Pauli exclusion principle leads to a reduction of shot noise from the Schottky value [2]. Coulomb interactions also act in correlating wavepackets, yet the Coulomb interactions may decrease or increase noise correlations [3]. Thus, in a given mesoscopic structure, the effects on the shot noise of Fermi statistics and of interactions are intimately mixed. In contrast, we propose here that spin-resolved shot noise can unambiguously probe the effects of interactions [4]. In a nutshell, the Pauli principle acting only on electrons with the same spin, currents wavepackets carried by quasiparticles with opposite spins can only be correlated by the interactions. "Spin current noise" has received little attention before, and with a different purpose. For instance, spin shot noise was recently considered in absence of charge current [5], and the effect of a spin-polarized current on charge and spin noise was investigated [6]. Noise is also an efficient probe for testing quantum correlations in two-electron spin-entangled states [7].

In contrast, let us consider mesoscopic structures in which the average current is *not* spin-polarized, but where the currents carried by quasiparticles with different spins can be separately measured. First, consider a mesoscopic device made of a normal metal with non-interacting electrons, non magnetic terminals i, j. In absence of magnetic fields and spin scattering , the scattering matrix is spin-independent, $s_{ij}^{\sigma\sigma'} = \delta_{\sigma\sigma'} s_{ij}$. Then one verifies that the spin-resolved noise, defined as $S_{ij}^{\sigma\sigma'}(t-t') = \frac{1}{2}\langle \Delta I_i^{\sigma}(t)\Delta I_j^{\sigma'}(t') + \Delta I_j^{\sigma'}(t')\Delta I_i^{\sigma}(t)\rangle$ where $\Delta I_i^{\sigma}(t) = I_i^{\sigma}(t) - \langle I_i^{\sigma}\rangle$, is diagonal in the spin variables, $S_{ij}^{\sigma\sigma'}(\omega) = \delta_{\sigma\sigma'} S_{ij}(\omega)$. Thus, choosing an arbitrary spin axis $\mathbf{z}$, the total (charge) current noise $S_{ij}^{ch} = S_{ij}^{\uparrow\uparrow} + S_{ij}^{\downarrow\downarrow} + S_{ij}^{\uparrow\downarrow} + S_{ij}^{\downarrow\uparrow}$ and the *spin current noise* $S_{ij}^{sp} = S_{ij}^{\uparrow\uparrow} + S_{ij}^{\downarrow\downarrow} - S_{ij}^{\uparrow\downarrow} - S_{ij}^{\downarrow\uparrow}$, defined as the correlation of the *spin currents* $I_i^{sp}(t) = I_i^{\uparrow}(t) - I_i^{\downarrow}(t)$, are strictly equal. On the contrary, in presence of interactions, one expects that $S_{ij}^{\uparrow\downarrow} = S_{ij}^{\downarrow\uparrow} \neq 0$, or equivalently $S_{ij}^{sp} \neq S_{ij}^{ch}$.

Let us first consider a NS junction, where S is a singlet superconductor and N a normal metal. The scattering matrix coupling electron (e) and holes (h) in the metal is made of spin-conserving normal terms $s_{ee}^{\sigma\sigma}$, $s_{hh}^{\sigma\sigma}$, and Andreev terms $s_{eh}^{\sigma-\sigma}$, $s_{he}^{\sigma-\sigma}$ coupling opposite spins. The total zero- frequency noise $S^{ch} = \sum_{\sigma\sigma'} S^{\sigma\sigma'}$ is given at zero temperature by the well-known result [8] $S^{ch} = \frac{4e^3V}{\pi\hbar} Tr[s_{he}^{\dagger} s_{he}(1 - s_{he}^{\dagger} s_{he})]$. We have in turn calculated the spin-resolved correlations $S^{\sigma\sigma}$ and $S^{\sigma-\sigma}$, and found that they are exactly *equal*. As a result, for a NS junction, at $T = 0$, the spin current shot noise is strictly zero, $S^{sp} = 0$. The current correlation between electrons with opposite spins is $S^{\uparrow\downarrow} = S^{\uparrow\uparrow}$, therefore *positive*. This "bunching" of opposite spins carriers is an obvious consequence of the Andreev process, e. g. the transmission of singlets through the interface. It has been recently discussed in a three-terminal geometry [9].

Let us now consider a small quantum dot in the sequential transport regime, where repulsive correlations are instead expected. It is connected by tunnel barriers to normal leads L and R with potentials $\mu_{L,R}$, with $eV = \mu_L - \mu_R$ (Fig. 1). One assumes that $\max(eV, k_BT) >> \hbar\Gamma_{L,R}$ and that only one level of energy E_0 sits between μ_R and μ_L. The dot can be in three possible occupation states ($N = 0, 1, 2$) of the level (Fig. 1). $U(N)$ being the Coulomb energy for the

state N, $\Delta E^+_{L,R}(N) = E_0 - \mu_{L,R} + U(N+1) - U(N)$ is needed to add an electron to state N from leads L, R, and $\Delta E^-_{L,R}(N) = -E_0 + \mu_{L,R} + U(N-1) - U(N)$ is needed to remove an electron from state N towards L, R. Let us further assume that $\Delta E^+_L(0)$, $\Delta E^-_R(1) << -k_B T$, which implies that the transitions from $N = 0$ to 1 involve electrons coming only from L, and the transitions from $N = 1$ to 0 involve electrons going only into R. One allows the Coulomb energy to vary and consider the possibility of transitions from $N = 1$ to 2, only from L, e.g. $\Delta E^-_R(2) << -k_B T$. This describes the following situation : if $\Delta E^+_L(1) >> k_B T$, the transition to state $N = 2$ is forbidden and only two charge states $N = 0, 1$ are involved (Fig. 1a). If on the contrary $\Delta E^+_L(1) << -k_B T$, then the three charge states 0, 1, 2 are involved (Fig. 1b). This physical situation corresponds for instance to fixing the gate voltage such as $U(1) = U(0)$, and varying the ratio between $k_B T$ and the Coulomb excess energy $U(2) - U(1)$.

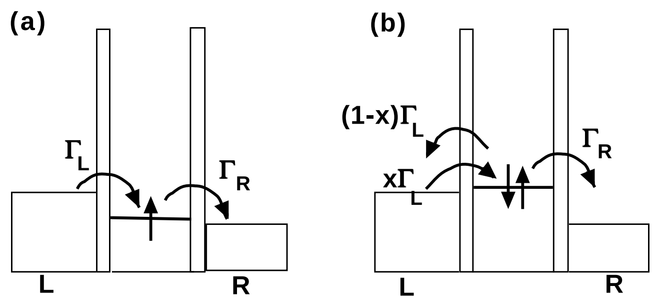

Figure 1: The SET transport sequence a) Between charge states $N = 0$ and 1 : rates Γ_L and Γ_R; b) Between charge states $N = 1$ and 2 : rates $x\Gamma_L$ from reservoir L, $(1-x)\Gamma_L$ to reservoir L and Γ_R to reservoir R.

Let us write the master equation describing this system [4]. Assuming a constant density of states in the reservoirs and defining x as the Fermi function $x = [1 + exp(\beta \Delta E^+_L(1))]^{-1}$, the populations p_0, $p_\uparrow$, $p_\downarrow$ and p_2 verify

$$\begin{aligned}
\dot{p}_0 &= -2\Gamma_L\, p_0 + \Gamma_R\,(p_\uparrow + p_\downarrow) \\
\dot{p}_\uparrow &= -(\Gamma_R + x\Gamma_L)\, p_\uparrow + \Gamma_L\, p_0 + ((1-x)\Gamma_L + \Gamma_R)\, p_2 \\
\dot{p}_\downarrow &= -(\Gamma_R + x\Gamma_L)\, p_\downarrow + \Gamma_L\, p_0 + ((1-x)\Gamma_L + \Gamma_R)\, p_2 \\
\dot{p}_2 &= -2((1-x)\Gamma_L + \Gamma_R)\, p_2 + x\Gamma_L\,(p_\uparrow + p_\downarrow)
\end{aligned} \tag{1}$$

Let us first consider the limit $x = 1$, corresponding to a resonant state without charging energy. Then spin $\uparrow$ and $\downarrow$ currents are uncorrelated, the average current is $\langle I\rangle = 2e\frac{\Gamma_L\Gamma_R}{\Gamma_L+\Gamma_R}$, the total zero-frequency noise [10] $S_{ij}(\omega = 0) = 2e\langle I\rangle(1 - \frac{2\Gamma_L\Gamma_R}{(\Gamma_L+\Gamma_R)^2})$. Here $S^{\uparrow\downarrow}_{ij} = S^{\downarrow\uparrow}_{ij} = 0$, or equivalently $S^{sp} = S^{ch}$. This is another example of uncorrelated transport.

Let us now consider the SET case $x = 0$, where charge transport is maximally correlated. The charge noise is given by $S_{ij}(\omega = 0) = 2e\langle I\rangle(1 - \frac{4\Gamma_L\Gamma_R}{(2\Gamma_L+\Gamma_R)^2})$ [11]. Apart from an effective doubling of the rate Γ_L, this result is qualitatively similar to that obtained without interactions. Therefore the charge noise is not the best possible probe of interactions. On the contrary, the behaviour of the spin noise is completely different. Using the method by Korotkov [12], we find that

$$S^{\sigma\sigma}_{ij} = e\langle I\rangle(1 - \tfrac{2\Gamma_L\Gamma_R}{(2\Gamma_L+\Gamma_R)^2}), \quad S^{\sigma-\sigma}_{ij} = -e\langle I\rangle\tfrac{2\Gamma_L\Gamma_R}{(2\Gamma_L+\Gamma_R)^2}, \qquad S^{sp}_{ij} = 2e\langle I\rangle \tag{2}$$

The result for S^{sp} resembles a Poisson result (maximal fluctuations). The correlations between currents of opposite spins are negative, like a partition noise. Yet spin-up and spin-down channels are separated as wavepackets with up or down spins exclude each other because of interactions, rather than statistics. Here, each junction is – due to Coulomb repulsion – sequentially crossed by elementary wavepackets with well-defined but uncorrelated spins. On the contrary, *charge* current wavepackets are correlated on times $\sim \hbar/\Gamma_i$, leading to the reduction as compared to the Poisson value. Notice that the analysis of the SET involving $N = 1$ and 2 states (instead of 0, 1) yields exactly the same result.

The general solution of Eqs. (1) spans the full regime between the uncorrelated and the maximally correlated cases. The average current is given by $\langle I \rangle = e\frac{2\Gamma_L\Gamma_R}{\Gamma_R+(2-x)\Gamma_L}$. The spin current noise components $S_{ij}^{\sigma\sigma'}$ (i,j=L,R) can also be calculated. The expression for the spin noise is $S_{ij}^{sp} = 2e\langle I\rangle\,(1 - \frac{2x\Gamma_L\Gamma_R}{(\Gamma_R+\Gamma_L)(\Gamma_R+x\Gamma_L)})$.

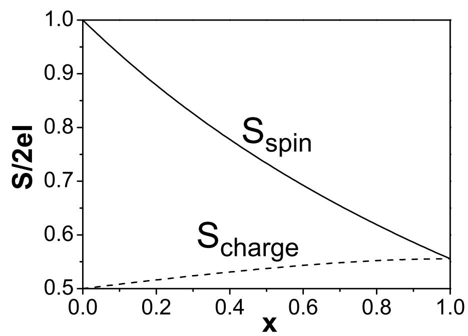

Figure 2: Spin shot noise and charge shot noise in the SET, as a function of x (see text) : $x = 0$ denotes the maximal correlation, $x = 1$ the uncorrelated case. $\Gamma_R = 2\Gamma_L$: antibunching of opposite spins.

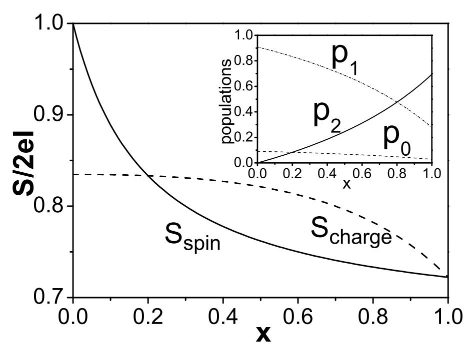

Figure 3: Same as Fig. 2, $\Gamma_R = 0.2\Gamma_L$: bunching of opposite spins for $x > x_c$. The inset shows the probabilities of states $N = 0, 1, 2$ and the population inversion at large x.

The expression for the total (charge) noise S^{ch} is too lengthy to be written here. Figs. 2, 3 show the variation with x of the charge and spin current noise. The spin noise is maximum for $x = 0$, decreases monotonously and merges the charge noise at $x = 1$. The role of the asymmetry of the junctions is very striking. First, if $\Gamma_R > \Gamma_L$, S^{sp} is always larger than S^{ch} (Fig. 2), like in the ideal SET ($x = 0$). On the contrary, if $\Gamma_R < \Gamma_L$, S^{sp} is smaller than S^{ch} for $x > x_c \sim \Gamma_R/\Gamma_L$ (Fig. 3). This implies that $S^{\uparrow\downarrow} > 0$, contrarily to the naive expectation for repulsive interactions : if $\Gamma_R < \Gamma_L$, the low charge states are unfavored and the high ones

favored, despite of Coulomb repulsion. Two electrons tend to enter the dot successively, with opposite spins, leading to a certain degree of bunching. Here the anomaly is due to a kind of "population inversion", manifesting a strong departure from equilibrium (Fig. 3).

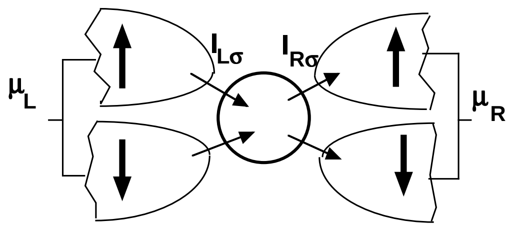

Figure 4: Schematic set-up for spin current measurement, using four spin-polarized terminals (see text).

Including a spin-flip rate $T_1^{-1} = \gamma_{sf}$, one finds $S_{LR}^{sp} = 2e\langle I\rangle \frac{\Gamma_R}{\Gamma_R+\gamma_{sf}}$, which suggests[4] a method to measure T_1. Fig. 4 shows a possible four-terminal set-up[13] for the measurement of spin current correlations, with ferromagnetic leads. In a fully symmetric device, the net current flowing through the SET is not spin polarized. Yet it is in principle possible to measure the noise correlations S_{L1L1}, S_{L1L2}, S_{L1R1}, S_{L1R2}, etc... If each terminal generates a fully spin-polarized current, the analysis of this set-up can be mapped onto the above model. If polarization is not perfect, the above measurement should mix spin noise with charge noise. If those are sufficiently different (strong repulsive correlations), they could still be distinguished, allowing to probe the Coulomb correlations by the method of spin current noise.

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